11.2 Videos Guide

11.2a

- Introduction to series
- Partial sums
  - $s_n = a_1 + a_2 + a_3 + \cdots + a_n$
- Sequence of partial sums
  - $\{s_n\} = s_1, s_2, s_3, \cdots, s_n$
- Convergence of a series
  - If the sequence $\{s_n\}$ is convergent and the limit $\lim_{n\to\infty} s_n = s$ exists as a finite real number then the associated series $\sum a_n$ is convergent, and the sum of the series is $\sum_{n=1}^{\infty} a_n = s$
- Convergence of a geometric series
  - $\sum_{n=1}^{\infty} a\, r^{n-1}$ is convergent if $|r| < 1$ and divergent otherwise. If $|r| < 1$, then $S = \frac{a}{1-r}$

11.2b

Theorem (statement and proof):

- If $\sum_{n=1}^{\infty} a_n$ is convergent, then $\lim_{n\to\infty} a_n = 0$.
- Divergence Test:
  If $\lim_{n\to\infty} a_n$ does not exist or if $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

11.2c

Exercises:

- Determine whether the series is convergent or divergent. If it is convergent, find its sum.
  $$\frac{1}{3} + \frac{2}{9} + \frac{1}{27} + \frac{2}{81} + \frac{1}{243} + \frac{2}{729} + \cdots$$
- Determine whether the series is convergent or divergent by expressing s_n as a telescoping sum. If it is convergent, find its sum.
  $$\sum_{n=1}^{\infty} \ln \frac{n}{n+1}$$

11.2d

Exercises:

- Find the values of x for which the series converges. Find the sum of the series for those values of x.

$$\sum_{n=1}^{\infty}(x+2)^n$$

- Express the number as a ratio of integers.
  $10.1\overline{35} = 10.135353535\ldots$
- A patient is injected with a drug every 12 hours. Immediately before each injection the concentration of the drug has been reduced by 90% and the new dose increases the concentration by 1.5 mg/L.
  a) What is the concentration after three doses?
  b) If C_n is the concentration after the nth dose, find a formula for C_n as a function of n.
  c) What is the limiting value of the concentration?